$SU(3)$ breaking and baryon magnetic moments

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Abstract

We show that the magnetic moments of the octet baryons can be fitted to an accuracy of 1.5 % by a phenomenological Lagrangian in which $SU(3)$ breaking corrections appear only linearly. This is in contrast to conventional chiral perturbation theory in which corrections non-analytic in $SU(3)$ breaking dominate and tend to spoil the agreement with the data. Motivated by this observation, we propose a modified scheme for chiral perturbation theory that gives rise to a similar linear breaking of $SU(3)$ symmetry. (Pacs 13.40.Em, 14.20.-c, 11.30.Rd) (hep-ph/9602251)

The magnetic moments of the octet baryons were found to obey approximate $SU(3)$ symmetry a long time ago by Coleman and Glashow [1]. In the $SU(3)$ symmetric limit, the nine observable moments (including the transitional moment between Σ^0 and Λ) can be parameterized in terms of two parameters, and as a result obey approximate relationships. As we will show later, the two parameter result of Coleman and Glashow can in fact fit the observed magnetic moments up to about the 20% level. However, since at present the moments have been measured with an accuracy of better than 1% [2], an improved theoretical understanding is clearly desirable. It is the goal of this paper to show how this can be achieved easily phenomenologically and in a scheme of chiral perturbation theory.

Many attempts were made trying to improve the numerical predictions of Coleman and Glashow by including the $SU(3)$ breaking effects using chiral perturbation theory (ChPT) [3, 4, 5, 6]. However, many of these efforts resulted in numerical fits *worse* than the leading order $SU(3)$ invariant one by Coleman and Glashow. For example, Caldi and Pagels [3] found that the leading $SU(3)$ breaking corrections, in their scheme for ChPT, appear in the non-analytic forms of $\sqrt{m_s}$ and $m_s \ln m_s$. They showed that the $\sqrt{m_s}$ corrections are in fact at least as large as the $SU(3)$ invariant zeroth order terms, which casts doubt on the applicability of ChPT. Caldi and Pagels suggested that this "failure" of ChPT might be attributed to the large mass of kaon in the loops and the fact that the leading correction is of non-analytic form. Such non-analytic contributions were indeed pointed out earlier by Li and Pagels [7] and others [8], however, the non-analyticity appears only in the $SU(3)$ invariant

chiral symmetry breaking mass, not in $SU(3)$ breaking parameters. More recently, similar large corrections to the baryon magnetic moments non-analytic in m_s have been found, by calculating them up to the one-loop level in ChPT [4, 5, 6]. By only using the $\sqrt{m_s}$ terms Jenkins *et al.* [5] could improve the accuracy of the Coleman-Glashow results from 20 % to about 10 %. However, this could only be achieved by using in kaon loops a *different* value of the meson decay constant than in pion loops, with the effect that the magnitude of the kaon loops is artificially reduced. In addition, Krause [4] showed that $m_s \ln m_s$ corrections are just as important, which disagrees with Refs. [5, 6]. Also, Krause further argued that the non-analytic contributions are not a good approximation of the loop integrals at all.

In the light of the problems of ChPT in accounting for the magnetic moments, we reconsider in this paper Okubo's extension [9] of the phenomenological approach of Coleman and Glashow for fitting the current accurate magnetic moment data. Motivated by our successful numerical results we discuss how ChPT can be formulated to avoid corrections non-analytic in m_s.

Following Ref. [9], we simply assume that the operators which give the leading $SU(3)$ breaking corrections to the magnetic moments have the same chiral transformation property as the strange quark mass operator, and demand that the coefficients of these operators should be of the order of m_s/Λ_χ (m_s is strange quark mass, and Λ_χ is chiral symmetry breaking scale).

To zeroth order in m_s/Λ_χ and to first order in electromagnetic coupling, the $SU(3)$ invariant terms that contribute to the magnetic moment can be written in a simple form as

$$b_1 \mathrm{Tr}\Big[\,\bar{H}\hat{O}\{Q,H\}\,\Big] + b_2 \mathrm{Tr}\Big[\,\bar{H}\hat{O}[Q,H]\,\Big]\ , \tag{1}$$

where the operator $\hat{O}$ is defined as $\hat{O} = F_{\mu\nu}\sigma^{\mu\nu}$, H is the usual representation for the baryon-octet in flavor-space given by Ref. [10], and Q is the quark charge-matrix

$$Q = \frac{1}{3}\mathrm{diag}(2,-1,-1)\ . \tag{2}$$

The leading-order results of Coleman and Glashow are based on the above form of the electromagnetic interaction. As shown in Table I, with b_1 and b_2 one can already account for the observed baryon magnetic moments at about the 20% level. To analyze the magnetic moments with the leading (linear) order of $SU(3)$ breaking corrections included, one needs to pay attention only to the flavor structure of the operators. Since the operator $\hat{O}$ is flavor independent, we will suppress it in the following.

We will assume, analogously as Okubo [9], that the lowest order $SU(3)$ breaking corrections are parameterized by baryon operators that have the same chiral transformation property as the quark-level strange quark mass operator. The operators in flavor space that are of first (linear) order in $SU(3)$ breaking can be constructed using the matrix $m_s\sigma$, where $\sigma = \mathrm{diag}(0,0,1)$. They include four single-trace terms

$$\begin{aligned}&\alpha_1 \mathrm{Tr}\Big[\,\bar{H}[[Q,H],\sigma]\,\Big] + \alpha_2 \mathrm{Tr}\Big[\,\bar{H}\{[Q,H],\sigma\}\,\Big]\\ &+ \alpha_3 \mathrm{Tr}\Big[\,\bar{H}[\{Q,H\},\sigma]\,\Big] + \alpha_4 \mathrm{Tr}\Big[\,\bar{H}\{\{Q,H\},\sigma\}\,\Big]\ ,\end{aligned} \tag{3}$$

and four double-trace terms

$$\begin{aligned}
&\beta_1 \mathrm{Tr}\Big[\,\bar{H}H\Big]\,\mathrm{Tr}\Big[\,\sigma Q\Big] \;+\beta_2 \mathrm{Tr}\Big[\,\bar{H}[Q,H]\Big]\,\mathrm{Tr}\Big[\,\sigma\Big] \\
&+\beta_3 \mathrm{Tr}\Big[\,\bar{H}\{Q,H\}\Big]\,\mathrm{Tr}\Big[\,\sigma\Big] \\
&+\Big(\beta_4 \mathrm{Tr}\Big[\,\bar{H}Q\Big]\,\mathrm{Tr}\Big[\,\sigma H\Big] + \mathrm{h.c.}\Big)\;.
\end{aligned} \tag{4}$$

The term with β_1 is diagonal in baryon space, i.e., contributes equally to any baryon magnetic moment, but not to the $\Sigma \to \Lambda$ transitional moment. Operators associated with β_2 and β_3 will not change the predictions based on b_1 and b_2, and can be ignored here. The operator associated with $\mathrm{Re}(\beta_4)$ can, by Cayley's theorem, (see Ref. [10]) be related to the other operators in Eqs. (4) and (5), while the operator associated with $\mathrm{Im}(\beta_4)$ is time reversal non-invariant and should be negligible in our analysis. Therefore, we only have to include α_1, α_2 α_3, α_4, and β_1 in our analysis.

Similar results can be found in the context of ChPT at tree-level. For example, Eqs. (1–4) also appear as counterterms in the chiral Lagrangians of Refs. [4, 5]. However, in contrast to ChPT in Refs. [4, 5], in our approach the flavor structure of Eqs. (1–4) is assumed to remain valid to *all* orders in momentum expansion.

We fit the eight available magnetic moment data with the seven parameters while requiring the resulting α_i, and β_1 to be a factor of order m_s/Λ_χ smaller than b_i for consistency. The resulting magnetic moments of baryons are given in the Table . Note that the sign of the transition moment between Σ^0 and Λ is a matter of convention. The average deviations of the fitted to the observed moments is 1.5 %, as compared to 20 % for the leading order fit.

At the first sight, a seven parameter fit of eight observables may not sound as much of an achievement. However, the fact that the typical values for α_i and the two parameters β_i turn out to be smaller than b_i by roughly a factor of m_s/Λ_χ is quite nontrivial and should be consider an evidence that the $SU(3)$ breaking effect should appear linearly in any model for magnetic moments of baryons. It is in sharp contrast with the ChPT in Refs. [4, 5] which yield corrections which are non-analytic in m_s and are of the same size of, or larger than, the leading-order ($SU(3)$ invariant) terms.

In addition, our result also predicts the not yet measured magnetic moment of Σ^0 to great accuracy. Based upon our fit, we expect $\mu_{\Sigma^0}/\mu_N = 0.66 \pm 0.03$ (where μ_N is the standard nuclear magneton) compared to $0.54{\pm}0.09$ from the lattice calculation of Leinweber, Woloshyn and Draper [11]. The magnetic moment of Σ^0 may continue to be difficult to measure directly, however, knowing its value accurately can be very useful both theoretically and experimentally elsewhere, such as weak radiative decays of hyperons.

Another outstanding issue [12] in the understanding of magnetic moments is the difference between the p and Σ^+ magnetic moment (which is vanishing in the $SU(3)$ symmetric limit). The fact that this difference can be accounted for satisfactorily by our current scheme is another support for linear $SU(3)$ breaking that we assumed.

The assumption that $SU(3)$ is broken linearly leads to the Okubo relations between the

magnetic moments [9]

$$\mu_{\Sigma^0-\Lambda} = \frac{1}{2\sqrt{3}} \left(\mu_{\Sigma^0} + 3\mu_\Lambda - 2\mu_{\Xi^0} - 2\mu_n \right) \ , \tag{5}$$

and

$$\mu_{\Sigma^0} = \frac{1}{2} \left(\mu_{\Sigma^+} + \mu_{\Sigma^-} \right) \ , \tag{6}$$

where $\mu_{\Sigma^0-\Lambda}$ is the transition moment between Σ^0 and Λ. The first relation agrees well with experiment, while the second can not be verified yet.

To put our result in proper perspective, one should be reminded that so far none of the existing models, e.g. such as those derived from the quark-model [13], have been able to reproduce the high precision data on the magnetic moments. On the other hand, none of the models have employed seven free parameters. For example, while Ref. [5] may not have accounted for the magnetic moment data successfully, the paper employs only the two $SU(3)$ invariant parameters b_1 and b_2. In their approach the $SU(3)$ breaking is only due to the meson masses in the loops. Our result should be taken as a strong indication that we need a scheme or model in which the leading $SU(3)$ breaking effects appears as the linear correction of order m_s/Λ_χ.

In the following we will discuss how one can modify the scheme of a ChPT so that a polynomial expansion in m_s is a result. One way to interpret the success of our phenomenological analysis is that the non-analytic contributions in m_s are actually not present, even in the context of ChPT. Such a scheme was already pointed out in the early works on ChPT, but it was abandoned in later applications. In the early papers of Li and Pagels [7] and Langacker and Pagels [8] the starting point to study deviations from chiral symmetry was the chiral symmetry breaking Hamiltonian

$$\epsilon H = \epsilon_0 H_0 + \epsilon_8 H_8 \ , \tag{7}$$

with the Hamiltonian H_0 invariant under $SU(3)$ but not under $SU(3) \times SU(3)$, and the Hamiltonian H_8 breaking both $SU(3)$ and $SU(3) \times SU(3)$. As the relevant parameters to measure the $SU(3) \times SU(3)$ and $SU(3)$ breaking

$$\epsilon_0 \propto M_\pi^2/\Lambda_\chi^2 \tag{8}$$

and

$$\epsilon_8 \propto \left(M_K^2 - M_\pi^2 \right)/\Lambda_\chi^2 \tag{9}$$

were proposed [8]. Note that the parameter ϵ_8 is proportional to m_s (assuming for simplicity that $m_u, m_d << m_s$). These authors showed that infra-red divergences caused by Goldstone bosons in the chiral loops will give rise to non-analytic terms in ϵ_0. However, they also pointed out that an expansion in the $SU(3)$ breaking parameter ϵ_8, still remains possible.

In ChPT this can be realized [14] by taking a $SU(3)$ invariant infra-red cut-off, denoted by M, for the pseudo-scalar mesons in the loops. In this scheme all the octet mesons have the propagator

$$S(k) = \frac{i}{k^2 - M^2} \ . \tag{10}$$

Effects due to breaking of $SU(3)$ can then be consistently treated perturbatively. When the scheme is applied to the magnetic moments, it means that all the $SU(3)$ breaking effects will remain polynomial in ϵ_8 (or equivalently m_s/Λ_χ) and the experimentally favored symmetry breaking pattern given by Eqs. (1), (3) and (4) remains valid to *all orders* in the loop expansion. Because M appears in the denominator of the meson propagators, loop diagrams will still give contributions non-analytic in M. However, since loop contributions will have the same symmetry structure as those Eqs. (1), (3) and (4) up to linear order in m_s, these non-analytic terms can be absorbed in the coefficients b_i, α_i, β_i. Our phenomenological analysis of the magnetic moments will be a natural consequence of this revised scheme of ChPT to linear order in m_s/Λ_χ.

To conclude, we showed that the experimental data for baryon magnetic moments seem to support a scheme of ChPT in which the $SU(3)$ breaking effect can be written as a polynomial expansion in m_s. This new scheme clearly differs from the schemes of ChPT used in Refs. [4, 5, 6] for the baryon magnetic moments. In those approaches both the pion mass and the kaon mass are used as the infra-red cut-off for their respective propagators in the loops. This necessarily leads to $SU(3)$ breaking effects different from that in Eqs. (1), (3) and (4) and furthermore to terms non-analytic in m_s. Since the interaction vertices are expanded perturbatively in m_s, it is not consistent to use m_s as propagator mass in the loops. In addition, our analysis shows that the contributions non-analytic in m_s are not needed to get a satisfactory fit with the data.

This work was supported by the National Science Council of the R.O.C. under Contracts Nos. NSC85-2112-M-007-032, NSC85-2112-M-007-029, and NSC85-2112-M-001-026.

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B	$\mathcal{M}_{\rm th}/e$	$\mathcal{M}_{\rm exp}/\mu_N$	$\mathcal{M}_{\rm SU(3)inv}/\mu_N$	$\mathcal{M}_{\rm SU(3)breaking}/\mu_N$
p	$b_1/3+b_2+\alpha_1+\alpha_2+(\alpha_3+\alpha_4)/3-\beta_1/3$	2.793 ± 0.000	2.29	2.793
n	$-2b_1/3-2(\alpha_3+\alpha_4)/3-\beta_1/3$	-1.91 ± 0.000	-1.48	-1.969
Λ	$-b_1/3-8\alpha_4/9-\beta_1/3$	-0.613 ± 0.004	-0.74	-0.604
Σ^+	$b_1/3+b_2-\beta_1/3$	2.458 ± 0.010	2.28	2.481
Σ^0	$b_1/3-\beta_1/3$	—	0.74	0.66
Σ^-	$b_1/3-b_2-\beta_1/3$	-1.160 ± 0.025	-0.80	-1.155
Ξ^0	$-2b_1/3+2(\alpha_3-\alpha_4)/3-\beta_1/3$	-1.250 ± 0.014	-1.48	-1.274
Ξ^-	$b_1/3-b_2+\alpha_1-\alpha_2-(\alpha_3-\alpha_4)/3-\beta_1/3$	-0.6507 ± 0.0025	-0.80	-0.6507
$\Sigma^0-\Lambda$	$b_1/\sqrt{3}$	$\pm 1.61\pm 0.08$	1.28	1.541

Table 1: Magnetic moments of the octet baryons, and transition moment for $\Sigma^0 \to \Lambda + \gamma$, with linear $SU(3)$ breaking corrections. The first column contains our predictions, with the constants b_i from Eq. (1), and the constants α_i, β_1 from Eqs. (3) and (4). The second column contains the experimental values from Ref. [2], and the third the fitted values in the $SU(3)$ symmetry limit. Finally, the fourth column contains the fitted values based on our symmetry prediction in column one. In units of GeV^{-1} we find $b_1 = 1.42$, $b_2 = 0.97$, $\alpha_1 = 0.32$, $\alpha_2 = -0.14$, $\alpha_3 = 0.28$, $\alpha_4 = -0.31$, and $\beta_1 = 0.36$. The average deviation of the predicted values from the observed is 1.5 %, while in the $SU(3)$ symmetric fit (see column three) this deviation is 20 %. We didn't weight the deviation of each fit value from the experimental value by the experimental standard deviation since the expected theoretical error is generally much larger than the experimental error bar.